

What am I?

A Game Connecting Mathematics and the Environment

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Teacher's Guide: Intermediate level

Lesson 3: *Applying the Mathematics of Food Webs*

Introduction

In this lesson, your students will investigate the mathematics of food webs more deeply. Understanding the mathematical structure of food webs can help students develop a greater respect for the complex and delicate balance of nature. Students will discuss ways in which their own activities affect animal food webs and, more generally, ways in which the balance of nature may be upset when people fail to think about the environment in a scientific way.

The lesson concludes with a game called “What Am I?” in which students can apply all of the things they learned about food webs.

Why the Math in Food Webs Matters

Start by reviewing the concepts about food webs that you covered in the last lesson. Then ask your students to discuss how these concepts might be relevant to the environment. Some questions that you might use to guide the discussion are provided below.

Remind students that “vertex” is just a mathematical name for a dot or point. A vertex on a graph can represent anything: for a biologist studying a food web, it can represent an animal; for an airline scheduler, it can represent a city that the airline flies into or out of; for a computer programmer, it can represent a task that the computer must do in a certain order; and for a sports scheduler, it can represent a game that must be played before or after another game. A “directed edge” on a graph is just an arrow pointing from one vertex to another. In a food web, the arrows or directed edges always point from the food to the animal that eats the food. You might put two versions of the same food web on the board (one with animal cards and one with vertices representing the animals) that you can refer to in the discussion.

1. What does the number of arrows (or edges) pointing towards or away from a vertex on a food web tell you, and how is this information important?

The number of edges pointing to a vertex representing an animal tells you how many animals that animal eats. The number of edges pointing away from a vertex tells you how many animals eat that animal.

An animal with many arrows pointing away from it provides energy directly to many other animals in the web. As a result, changes in the environment which affect that animal also affect many other animals in the web.

An animal that has many arrows pointing towards it is one that consumes many other animals. These kinds of animals can be very helpful to humans. For instance, owls consume a great variety of rodents (rats, mice, shrews, and voles) and insects that eat farmers' crops. Barn owls can be more effective at controlling the various animals that farmers consider pests than pesticides or poisons. Having a family of barn owls on a farm is much healthier for the environment than having a barn or field full of potentially harmful chemicals.

2. Why is the length of the longest path in a food web important?

All animals ultimately depend on the energy of the sun that is stored in plants. At each level of the food web, much of the energy that can be used to sustain life is lost. You can get an idea of how much energy is lost if you look at an owl pellet. Recall from the last lesson that you can find entire skeletons of small

animals in owl pellets, as well as hair and other indigestible materials. All of this matter, which may have come from a mouse or a frog, contains energy that the mouse or frog absorbed from smaller creatures but that the owl can't use. So you can see how much energy is lost at each level in the food chain. That's why several acres of forest or meadow can support only a few animals at the top levels of a food chain. Therefore, to protect animals like owls, hawks, lynxes, and bears, humans must preserve large areas of wilderness and keep them free from pollution and development.

(Later in the lesson your students will learn to calculate how much energy is lost at each level in a food chain, and they will discover some surprising implications about how and what they should eat if they want to protect the environment).

There is another reason why long food chains are important to humans. Because so much energy is lost at each level, animals that are at the top of the food chain must consume large masses (relative to their body weight) of the animals that are below them.

Barn owl babies can eat their own weight in food each day. How many kilograms of food would you have to eat in a day to eat your own weight? About how many kilograms do you normally eat in a meal? Probably not more than one or two. How many meals would you have to eat in a day to eat your weight in food? Can you estimate how much time you would have to spend eating each day to consume food at the same rate as a baby barn owl? When would you find the time to do your homework?

If the animals and plants at the bottom of a food chain absorb any harmful chemicals from pollution or pesticides, these chemicals will become more and more concentrated in animals' bodies as you move up the food chain (because animals high in the chain consume so much of what is below them). Even chemicals that are weakly concentrated in the bodies of small animals and plants can pose a threat to the people and animals who eat those animals and plants. That is why, for instance, the government warns people not to eat fish from some of the Great Lakes, because the chemicals that we put into the water have become so concentrated in those fish. The longer a food chain is, the more concentrated the chemicals that humans introduce into the environment will be.

3. Why is it important to know what vertices you can reach on a graph by following a path from a particular vertex?

If you start at a particular animal on a web and follow all the paths that lead away from that animal, you find all the animals that depend on the animal for their food energy. If the animal is harmed by human activities, then all the animals in the web can suffer. Following paths on a food web will also tell you where the chemicals that an animal eats or absorbs will become concentrated.

The frog is one of the most important animals in many Canadian food webs, because many other animals, like fish and owls, depend on it for food. But the frog is also one of the Canadian animals that is most vulnerable to changes in the environment, because it is an amphibian. Amphibians like frogs and salamanders develop from eggs laid in water. When they are young they breathe water through gills, like fish do, and when they are older they develop lungs and breathe air, like we do. (Imagine spending your childhood breathing water and then learning to breath air!) Amphibians have extremely sensitive skin that easily absorbs the pollution and pesticides that people put in water. Populations of amphibians have declined all around the world because the marshes and ponds they depend on are being polluted or developed by humans. As a result, animals that are higher up in food webs have less to eat. (Frogs are also important because they eat vast numbers of insects. The fewer frogs there are, the more people rely on pesticides to control insects).

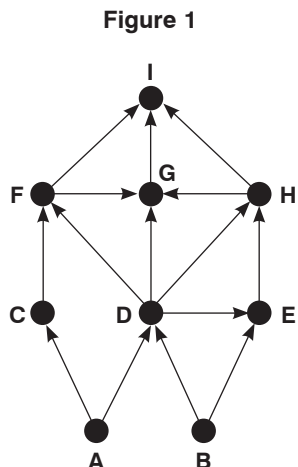
4. If you remove enough vertices from a graph, you can "disconnect" it so that a set of the remaining vertices cannot be reached from another set. Why is it important to know which sets of vertices will disconnect a graph when they are removed?

Suppose there is a set of animals that disconnect a food web when you remove them. If those animals were to become extinct, then all the animals that are above them in the food web would also become extinct (unless they had some other sources of food), because they would become disconnected from

the energy of the sun that is passed through the food web.

Tell your students that you will now give them a chance to develop their understanding of food webs by identifying all of the structures you have discussed. This exercise will help them play the game that you are saving for the end of the lesson.

Draw the graph in Figure 1 on the board and ask your students the following questions.



How many edges point towards vertex D? (Two). How many point away? (Four). How long is the longest path on the graph? (Six vertices long). How many paths are there of this length? (One path: B, D, E, H, G, I). How many paths can you find from the bottom to the top of the graph that are four vertices long? (Warn your students that this is a tricky question. To find all the paths they will have to work systematically, starting at one of the bottom vertices and not moving on until they have found all paths from that vertex to the top of the graph. There are eight paths altogether). Which vertices can you reach from vertex C? (F, G, and I).

Give your students a copy of BLM 2: Directed Graphs (Intermediate). Ask them to answer the following questions in a notebook. Students needn't answer every question for every graph; you might give students a fixed amount of time to work on the graphs, but don't make it a competition to see how many they can finish.

- How many edges point towards vertex F?
Answers: For G1, two edges; G2, three edges; G3, one edge; G4, one edge
- How many edges point away from vertex F?
Answers: For G1, no edges; G2, one edge; G3, three edges; G4, one edge
- What is the length of the longest path?
Answers: For G1, 5 vertices long; G2, 5 vertices; G3, 6 vertices; G4, 6 vertices
- How many longest paths can you find?
Answers: For G1, 1 path; G2, 2 paths; G3, 5 paths; G4, 2 paths
- Which vertices can you reach from vertex C?
Answers: For G1, E, D, and F; G2, F and G; G3, E, H, and K; G4, F, I, K, L, H, and J
- What is the minimum number of vertices that you need to remove to disconnect the graph?
Answers: For G1, 2 vertices (For example, if you remove vertices B and C, and all of the edges that end or begin at B and C, then vertex A becomes disconnected from vertices D, E, and F); G2, 2 vertices (If you remove B and F then C becomes disconnected from the rest of the graph); G3, 1 vertex (If you remove B then A becomes disconnected); G4, 1 vertex (Remove E, F, or I and the graph becomes disconnected).

After students have completed the exercise above, tell them that you would like to look more closely at the environmental consequences of some of the concepts they have learned.

Scientists have determined that only a tenth of the energy that is stored at a particular level in the food chain is passed on in a form that is usable at the next level. This means that if a given area of land can support 1000 kilograms of plants, it can support only about 100 kilograms of animals that eat the plants, 10 kilograms of animals that eat those animals, and 1 kilogram of animals that eat those animals. Also, because animals at each level must eat such a great volume of animals at the lower levels, harmful chemicals become ten times more concentrated at each level.

Ask students to discuss some of the things that they do at home or at school that might introduce harmful chemicals or substances into animal food webs or that might affect animal habitats. They could make a list of some of the things they might do that would reduce their impact on the environment.

Here are some examples of topics you could discuss.

1. Only a tenth of the energy in grain is absorbed when an animal eats the grain. So to produce just 1 kilogram of usable energy in meat, how many kilograms of grain do we need to grow? (10 kilograms) If we eat 1 kilogram of grain directly (rather than the kilogram of meat), about how many kilograms of wheat do we save? (9 kilograms) How many kilograms of grain would be saved if you replaced only 10 kilograms of meat in your own diet with 10 kilograms of grains or vegetables? (90 kilograms, which can be rounded to 100 kilograms for the calculations below). Think about the weight of meat that you consume in an average meal. About how many meals would it take you to consume 10 kilograms of meat? How many times would you have to skip meat to save about 100 kilograms of grain? How many times to save a tonne? Could you save a tonne of grain in a year? As a research project, older students might investigate how much land would be required to grow that grain and how many kilograms of pesticides would normally be used on that land. Fact: U.S. livestock consumes about seven times as much grain as Americans do themselves. (Source: Inkling Magazine website).

(Make sure your students know that the numbers above are rough estimates and that the amount of energy saved in replacing meat with vegetables will vary).

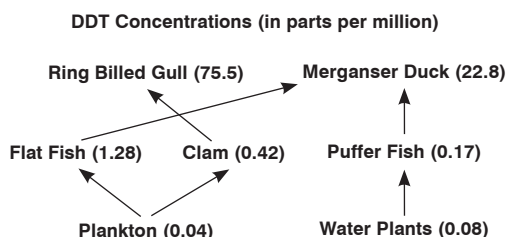
2. Pesticides are widely used in Canada to kill insects. The chemical DDT was popular in the 1960s but is now banned in North America because scientists eventually discovered that birds in the top levels of food webs had so much DDT in their bodies that they were laying eggs with extremely thin shells which would break before they hatched.

Many pesticides have long “half lives”. That means that it takes a long time before even half of the pesticide is broken down into a harmless form. DDT has a half life of 10 to 15 years. That means that if one tonne of DDT was sprayed on a particular area, 500 kilograms of the substance would still be present in the area 10 to 15 years later, and 250 kilograms would be present 20 to 30 years later. How much would be present 40 to 60 years later? (Half of 250 kilograms, which is 125 kilograms).

Pesticides with long half lives have been discovered as far away as the Arctic and Antarctic; they were carried there on soil particles by the wind.

Figure 2 shows some concentrations of DDT that were found in marine animals (Source: *Environmental Science*, E. Enger, B. Smith, WCB Publishers, Chicago, 1995). The amount of DDT in the animals is measured in parts per million. A part per million is a unit of measurement used to describe the concentration of substances in soil, water or tissue. If there is 1 part per million of DDT in an animal's tissue, that means that there is 1 milligram of DDT in every kilogram of tissue. Since there are 1000 milligrams in a gram, this is equivalent to saying that there would be 1 gram of DDT in 1000 kilograms (a tonne) of tissue. This may seem like a very small amount, but to see that it isn't, consider this: An elephant can weigh 40 tonnes. Imagine feeding an elephant 40 grams of poison!

Figure 2



Let your students know that concentrations of harmful chemicals don't always increase ten times as you move from one level in a food chain to another; this is just a rough estimate. Ask your students to check whether the concentration of DDT in the animals in the food web in Figure 2 actually increase by a factor of ten as you move up each level. To make it easier for students to check this, remind them that if they want to compare a pair of decimal numbers, they can multiply each number by a power of ten (that is, by 10, 100, 1000, etc.) to shift the decimal. For instance, to compare the numbers .17 (for the fish) and .08 (for the water plants), students could multiply each number by 100 to shift the decimal two places. Then they could compare the numbers 17 and 8 and see that the concentration of DDT increases by a factor of about two times between the plants and the fish (since 8 divides into 17 about two times).

Note: If your students don't know why the decimal shifts to the right when they multiply a number by 10 or 100, review the meaning of decimal notation. The decimal notation 0.1 (also written 0.10) stands for an amount that fits into a whole ten times. A dime fits into a dollar ten times, which is why a dime is written \$0.10 (the dollar is the whole). If you take an amount that fits into a whole ten times, and you multiply that amount by ten, you will get the whole. For instance, if you take ten dimes you will get a dollar, because by definition a dime fits into a dollar ten times. That is why when you multiply 0.1 by 10 you get 1.0 (or 1.00). Similarly, the decimal notation 0.01 stands for something that fits into the whole 100 times. A penny fits into a dollar 100 times, which is why it is written \$0.01. If you take an amount that fits into a whole a hundred times and multiply it by 100, you will get the whole. For instance, if you take 100 pennies you will get a dollar, because by definition a penny fits into a dollar 100 times. That's why when you multiply 0.01 by 100 you get 1.00.

Ask your students whether they can find an example in the chart where the concentration of DDT increases between levels by about ten times. (**Answer:** Yes, the rate of increase between the plankton and the clam is about ten times). Can they find an example where the concentration increases by more than ten times? (**Answer:** Yes, there are several examples. For instance, the concentrations for the clam and the ring billed gull are .42 and 75.5 respectively. Multiplying both numbers by 100 gives 42 and 7550. Rounding 7550 to the nearest thousand gives 8000 and 42 to the nearest ten gives 40. Dividing 8000 by 40 gives 200. So the rate of increase is almost two hundred times!)

3. Ask your students to discuss the ways they use water at home and at school and to list things they do that might impact the quality of the water for animals. Often the products that clean clothes, floors or dishes pollute the water. How can "cleaners" make water less clean?

Here is an example of how this happens. Many brands of laundry soap contain chemicals called phosphates. Phosphates cause a plant called "algae" (the green slime you see in dirty aquariums) to grow out of control in lakes and streams. In lakes where there is too much algae, fish and other aquatic animals can't survive. (That's because the small organisms that feed on the algae multiply and use up the oxygen in the water that fish need to live).

Fortunately, many stores now sell cleaners that don't contain phosphates and other harmful chemicals. (Your class might do some research on where they could find cleaners that don't harm the environment).

4. Challenge your students to think of other ways they can protect the environment by protecting animal food chains and habitats.

To finish the lesson, play the following game. Give each student a copy of BLM 4: A Food Web, or place several copies around the class where students can easily see it. Tape a copy of an animal card on each student's back. Students must find out what animal they are by asking other students questions that are related to the food web. They might ask, for instance: Am I a producer? Am I a consumer? Am I a herbivore? Am I a carnivore? Am I an omnivore? Am I a primary consumer? A secondary consumer? A tertiary consumer? Do I eat grasses? Do I derive my energy from a frog? (Before the game, remind your students that an animal "derives its energy" from another animal if there is a path of arrows in the web that leads from the other animal to the animal that depends on it). Students might write down their questions in a notebook and might also discuss the strategies they used in the game.

When the game is over, congratulate your students for the work they have done discovering so many ways in which living things are interrelated. And thank them for thinking so deeply about what they can do to preserve the complex and delicate balance of the food webs of the world.